GT Math 2403 T2 Equations Sheet

Chapter 2

(1) **General FOLDE** $\frac{dy}{dt} + p(t)y = g(t)$. Integrating Factor $u(t) = e^{\int p(t)\,dt}$

(2) **Logistic Growth** $\frac{dy}{dt} = r(1 - \frac{y}{K})y$ where r is growth rate and K is carrying capacity

(3) **Criteria for Exact Eqns** $M(x,y)dx + N(x,y)dy = 0$ is exact IFF $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(4) **Integrating Factor Depends Only on x?** then $\frac{M_y - N_x}{N}$ depends only on x switch N_x and M_y to test if integrating factor depends only on y.

Chapter 3: First Order Systems

(1) **FOLS Test for Unique Solution** If the coefficient functions of a general linear system are continuous on an open interval I, then there exists a unique solution to the system which exists throughout the interval for any given initial values.
(2) **Wronskian** If the Wronskian of two solutions is not zero, they form a fundamental set.
(3) **Real and Different Eigenvalues** $y(t) = c_1 e^{\lambda_1 t}\boldsymbol{v_1} + c_2 e^{\lambda_2 t}\boldsymbol{v_2}$
(4) **Complex Eigenvalues** Use Euler's formula to simplify the solution into its real part $u(t)$ and its complex part $w(t)$. The general solution then is $x = c_1 u(t) + c_2 w(t)$
(5) **Repeated Eigenvalues, One Eigenvector** Find w satisfying $(A - \lambda I)w = v$. An additional solution is then $e^{\lambda t}(w + tv)$

Chapter 4: Second Order LDEs

Find roots of characteristic polynomial. Then use below formulas to complete the solution.
(1) **Real and Distinct** $c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$
(2) **Complex** Roots of form $\lambda \pm \mu i$ then real part of solution is $c_1 e^{\lambda t}\cos\mu t + c_2 e^{\lambda t}\sin\mu t$
(3) **Repeated** $c_1 e^{\lambda_1 t} + c_2 t e^{\lambda_2 t}$
(4) **Spring Equation** $my''(t) + \gamma y'(t) + ky(t) = F(t)$
(5) **Damping** $c = \gamma^2 - 4km$. If $c < 0$ then underdamped. If $c = 0$ then critically damped. If $c > 0$ then overdamped
(6) **Undetermined Coefficients**

The particular solution of $ay'' + by' + cy = g_i(t)$...

$g_i(t)$	$Y_i(t)$
$P_n(t) = a_0 t^n + a_1 t^{n-1} + ... + a_n$	$t^s(A_0 t^n + A_1 t^(n-1) + ... + A_n$
$P_n(t)e^{\alpha t}$	$t^s(A_0 t^n + A_1 t^{n-1} + ... + A_n)e^{\alpha t}$
$P_n(t)e^{\alpha t}\begin{cases}\sin\beta t\\ \cos\beta t\end{cases}$	$t^s[(A_0 t^n + A_1 t^{n-1} + ... + A_n)e^{\alpha t}\cos\beta t + (B_0 t^n + B_1 t^{n-1} + ... + B_n)e^{\alpha t}\sin\beta t]$

(7) **Variation of Parameters 1** Given $[x_1, x_2]$ which form a fundamental set of solutions to $x' = P(t)x$, then $X = \{x_1, x_2\}$, and a particular solution to $y'' + p(t)y' + q(t)y = g(t)$ is $\boxed{X(t)\int X^{-1}(t)g(t)dt}$

(8) **Variation of Parameters 2** Given solutions y_1 and y_2 to the corresponding homogenous equation, a particular solution to $y'' + p(t)y' + q(t)y = g(t)$ is

$$Y(t) = -y_1(t)\int \frac{y_2(t)g(t)}{W[y_1, y_2](t)}dt + \int \frac{y_1(t)g(t)}{W[y_1, y_2](t)}dt$$

Chapter 5: Laplace Transformations

(1) **Definition of Laplace Transform** $\int_0^\infty e^{-st} f(t)dt$

Table of Common Laplace Transformations

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$	$e^{ct}f(t)$	$F(s-c)$
t^n	$\frac{n!}{s^{n+1}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\cosh at$	$\frac{s}{s^2-a^2}$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$	$\delta(t-c)$	e^{-cs}
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$	$f'(t)$	$sF(s)-f(0)$

Chapter 7: Nonlinear Differential Equations and Stability

(1) **Condition for Linear Approximation** Given general nonlinear system $x' = F(x,y)$, $y' = G(x,y)$ (A). The system is almost linear in the neighborhood of any isolated critical point (x_0, y_0) whenever the functions F and G have continuous partial derivatives up to order 2.
(2) **Almost Linear Approximation** Find the Jacobian matrix of the system. Plug in the x and y values of each critical point for a linear approximation of the system near that critical point.